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Suppose that $\chi(G) = k$ where $k \geq 1$

We cou categorise V into subsets of vertices colored in colours between 1 and k. lets maue those subsets $V_1 \dots V_k$.

Assume some colors i ~~i~~ and j which color respectively the subsets V_i and V_j.

Between each two subsets V_i and V_j there has to be at least one pair of vertices $v \in V_i$ and $u \in V_j$ such that v and u are adjacet (They are endpoints of some edge e) $\longrightarrow$ This is from the assumption that V_i and V_j contain differently colored elements. From this it follows that v and u have to be connected by an edge, otherwise there would be no reason for elements of V_i and V_j to be differently colored.

We know that $\chi(G) = k$ and all the k differently colored subsets have at least one edge between.
Therefore the lower bound on m nuben of edges is $\binom{k}{2} = \frac{k!}{(k-2)!\,2!} = \frac{k(k-1)}{2}$

Therefore we subtitele.

$k \leq \sqrt{2m} + 1$

$k \leq \sqrt{k(k-1)} + 1$

$k - 1 \leq \sqrt{k^2 - k} \quad |\,^2$

$k^2 - 2k + 1 \leq k^2 - k \quad | -k^2$

$-2k + 1 \leq -k \quad | +2k$

$1 \leq k$

This is always true from the assumption. Therefore we proved that $\chi(G) \leq \sqrt{2m} + 1$